A Heuristic Routing Algorithm for Network Coding Aware 1+1 Protection Route Design for Instantaneous Recovery

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***Abstract*—This paper proposes a heuristic routing algorithm to design instantaneous recovery protection routes for all possible source destination pairs by provisioning network coding (NC) based 1+1 protection technique. We consider a static routing problem in networks where each node has the coding capability, and the exact traffic demand matrix is given. In the proposed heuristic algorithm a network with N nodes is divided into N scenarios, where each node is chosen as the common destination and k nodes among the remaining ones are the sources, where $2 \leq k \leq N-1$. By dividing the network into several scenarios with k sources and a common destination (kSD), all the possible source destination pairs, according to the given traffic matrix, are considered. It was reported that a mathematical programming approach to determine NC based 1+1 protection routes for any kSD scenario is an intractable problem for large k values. In the proposed heuristic algorithm we tackle this intractable problem by choosing either two or three sources out of k sources at a time according to the largest effective gain first policy, and then routing is assigned to the selected $2SD$ or $3SD$ scenario by using our developed mathematical models. The largest effective gain first policy ensures the best possible resource saving for each of the selected $2SD$ or $3SD$ scenario. We compare the total path costs of NC based 1+1 protection for all possible source destination pairs, obtained by our proposed heuristic algorithm, with that of the conventional 1+1 protection technique (without NC). Numerical results observes that almost 15% resource saving is achieved in our examined networks.**

Keywords- **Routing, network coding, 1+1 protection, instantaneous recovery, integer linear programming, heuristic algorithm.**

I. Introduction

Network protection is a prominent research area, and several techniques for provisioning protection against network component failures have been innovated in recent years. These techniques are broadly classified as either predesigned protection or dynamic restoration techniques [1]. 1+1 protection is a simple predesigned protection technique for protection against any single network failure where there are two disjoint paths between every source and destination pair. Copies of the same data are sent simultaneously through these two disjoint paths. If one path fails, the destination node quickly switches over to the other path for receiving the data.

Reliable communications employing protection techniques have several service grades. The highest service grade is instantaneous recovery from failure, for which 1+1 protection is appropriate. The next service grade is to ensure protection against failure using less resources at the cost of increased recovery time. Shared backup path protection (SBPP) [2] and dynamic restoration [3] are resource efficient protection techniques to deal with single failure events. In these two techniques backup paths are shared to achieve resource efficiency. 1:1 protection does not allow backup sharing, but backup paths may be used to send low priority communications when active path is online.

In SBPP, and 1:1 protection, switching operations at least at both ends are required for the recovery operation, while in dynamic restoration switching operations are required at two nodes corresponding to the failed path or link after finding the recovered route. The necessity of switching at least at two nodes restricts instantaneous recovery of failed data. Thus these three techniques are not suitable to achieve our objective of instantaneous recovery.

1+1 protection provides instantaneous receiver-initiated recovery from any single link failure in the network. In order to achieve this, the resource requirements between every possible source and destination have to be doubled. Now-a-days traffic demands are increasing rapidly. In high capacity networks, failure protection is a necessary part, while it is desirable that provisioned protection technique requires less amount of network resources.

The network coding (NC) technique, first introduced in [4], increases network throughput using less amount of network resources. If NC is employed, in addition to forwarding, intermediate nodes are allowed to modify the incoming data. In the NC technique, the data stream on the output link from an intermediate node is the linear combination of the data streams on its input links. To be able to apply NC a node must have degree at least 3, i.e., at least two input data stream is required to apply NC and the resultant data stream is sent onto the output link of the node.

Literature reviews show that the NC technique is used for $1+N$ protection [5], [6], [7], against single and multiple link failures [8], [9], [10], for multicast protection [11], and for NC aware routing in wireless networks [12], [13]. In [5], [6], [7] 1+1 protection without NC is considered as the conventional method of protection, and is compared to NC based $1+N$ protection. In other works 1+1 protection with NC was not considered.

The issue that the NC technique is able to reduce the resource utilization in the resource hungry 1+1 protection, in scenarios with two sources and a common destination ($2SD$), was first addressed in [14]. In order to determine an optimum set of NC aware 1+1 protection routes for $2SD$ scenarios, an

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integer linear programming (ILP) model was presented in [15]. It was also mentioned in [15] that the ILP approach for $2SD$ scenarios can be extended to solve NC based 1+1 protection routing in any scenario with $k \geq 3$ sources and a common destination.

A. Issues to be addressed

There are two issues that were not addressed in a prior work [15]. One, NC based 1+1 protection route design for the whole network was not addressed. The resource saving reported in [15] considered several arbitrarily chosen $2SD$ scenarios only. Two problems can arise due to arbitrary choice. First, the same source node can appear in multiple $2SD$ scenarios considered. Second, it may not ensure the best possible resource saving when multiple connections having a common destination are served by selecting two sources arbitrarily at a time. Two, no mathematical model for scenarios with $k \geq 3$ sources and a common destination was presented in [15]. Only 13 possible NC situations for $3SD$ scenarios were presented in [15].

B. Our contributions

Our contributions consist of two parts. In the first part, we present ILP models for all the 13 NC situations for $3SD$ scenarios. In the second part, we propose a heuristic routing algorithm to design instantaneous recovery protection routes for all possible source destination pairs by provisioning network coding (NC) based 1+1 protection technique. Note that the ILP models presented in the first part are used in the proposed heuristic algorithm.

First, we describe a systematic analysis procedure for deriving mathematical models for the $3SD$ scenarios, where 4 XOR operations are considered for the NC operation. This procedure results in a framework of 13 mathematical models. Using our systematic analysis procedure it is possible to derive a mathematical model framework for $kSD, k \geq 4$ scenarios.

Second, we propose a heuristic routing algorithm to implement NC based 1+1 protection routes for all possible source destination pairs in the network. In the proposed algorithm a network with N nodes is divided into N scenarios, where each node is chosen as the common destination and k nodes among the remaining ones are the sources, where $2 \leq k \leq N{-}1$. For each of the N divided scenario with k sources and a common destination (kSD), we choose either two or three sources out of k sources at a time according to the largest effective gain first policy, and then routing is assigned to the selected $2SD$ or $3SD$ scenario by using our developed mathematical models.

The remainder of this paper is organized as follows. Section II introduces a systematic analysis procedure for analyzing various NC decision situations for $kSD, k \geq 3$, scenarios with the scenario classifications and descriptions. This analysis procedure is described for $3SD$ scenarios. Section III uses a network model to introduce the terminologies of this paper. Section IV illustrates the mathematical models for $3SD$ scenarios, which are derived from the analysis procedure. Section V describes the proposed heuristic routing algorithm based on the largest effective gain first policy. Section VI evaluates the performance of the proposed algorithm in terms of the total path cost. Finally, section VII summarizes the key points.

II. Systematic analysis procedure

The systematic analysis procedure for analyzing any kSD scenario, where $k \geq 3$, consists of three parts, which are described in the following:

- Classify the scenarios considering all possible combinations of XOR operations, and specify for each scenario that how many paths carry plain data, and how many carry encoded (NC) data.
- Analyze the scenarios properly to determine the specific paths that are subject to NC. Because path disjoint constraints are not imposed among these paths. Note that, for a kSD scenario, there are $2k$ paths, and $\binom{2k}{2}$ possible path disjoint constraints to ensure that $2k$ paths are mutually disjoint.
- Specify the objective function for every possible scenario considering the selected paths that are subject to NC, and construct the mathematical model. The flow conservation constraints for all the possible scenarios are the same.

The above procedure is illustrated with respect to the $3SD$ scenarios below.

A. Scenario classification

The $3SD$ network configurations are classified into five scenarios. From scenario 2 to scenario 5, each has three sub-scenarios, yielding a total of 13 sub-scenarios. The scenarios and sub scenarios, presented in Fig. A·2 of [15], are described as follows:

- Scenario 1: Among the six paths, no path is subject to NC, i.e., non-NC scenario.
- Scenario 2: Three backup paths from the three sources carry network coded data.
- Scenario 3: Among the six paths, four paths experience the NC effect in pairs.
- Scenario 4: Only two paths from any two sources are network coded, where common destination node's degree is three.
- Scenario 5: Only two paths from any two sources are network coded, where common destination node's degree is five.

B. Determine the path disjoint constraints and the objective functions

One question arises: *How path disjoint constraints are assigned?* We answer this question with respect to the scenario shown in Fig. 1. In this scenario, for six paths $P_{sm}, s = 1,2,3, m = 1,2$ to be mutually disjoint $\binom{2\times 3}{2} = 15$ path disjoint constraints are needed . As shown in the Fig. 1(b), the paths indexed by P_{12}, P_{21}, and P_{31} experience the NC effect. Therefore, they need not be mutually disjoint. That is why we *do not impose* the following three path disjoint constraints in the mathematical model for this particular scenario:

- P_{12} and P_{21} are not disjoint.

- P_{12} and P_{31} are not disjoint.
- P_{21} and P_{31} are not disjoint.

Then the objective function is assigned correctly considering the selected constraints to get the proper NC effect.

The objective functions and the path disjoint constraints for all the 13 sub-scenarios are specified in Table I of Appendix A. The objective function in NC cases contains two or three expressions. The first part gives the cost for provisioning 1+1 protection, the second (and third term in some cases) term accounts for the NC saving. The flow conservation constraints in all the above mentioned 13 sub-scenarios are the same.

III. Network model

The network is represented as a directed graph $G(V, E)$, where V is the set of vertices (nodes) and E is the set of links. There are N nodes and $|E|$ links in $G(V, E)$. A link from node $i \in V$ to node $j \in V$ is denoted as $(i, j) \in E, i \neq j$. $x_{ij}^{sd,m}$ is the binary routing variable. If any link $(i, j) \in E$ belongs to the disjoint path number m between source node s and destination node d, then value of $x_{ij}^{sd,m}$ is 1, otherwise 0, where m is either 1 or 2. c_{ij} is the cost of $(i, j) \in E$. It is assumed that the traffic demands for all possible source destination pairs are equal. The network is bi-connected, i.e., it is ensured that in the network, for every possible source-destination pair, there exist two disjoint paths for applying 1+1 protection. Every node with degree at least three has the NC capability, but encoded data are only decoded at the destination node. P_{sm} indicates the mth path between source node s and common destination node d, where m is either 1 or 2. Let $\mathcal{S}$ be a set of vectors (a, b, c, e), where a and c are source node indexes, and b and e are path indexes having a value either 1 or 2. a and c can have any integer value from 1 to k, where k is the number of source nodes having a common destination. Each vector $(a, b, c, e) \in \mathcal{S}$ corresponds to a path disjoint constraint for the scenario under consideration. $\mathcal{S}$ is an input parameter to the mathematical model.

NC gain, denoted by G_{NC}, implies that how much network resources is saved in 1+1 protection by using the NC technique. Let us assume that ξ_{NC} indicates the cost of employing 1+1 protection (in a scenario with two or more sources and a common destination) with NC and ξ_{NO_NC} indicates the same cost without NC effect. G_{NC} is defined by,

$$G_{NC} = \max\left\{\frac{\xi_{NO\_NC} - \xi_{NC}}{\xi_{NO\_NC}}, 0\right\}. \quad (1)$$

Equation (1) states that if the cost with NC is smaller than that of without NC, we achieve some positive gain. Otherwise there is no (or zero) gain and the solution without NC is adapted.

IV. Mathematical formulations for the $3SD$ scenarios

This section describes how the mathematical programming formulations are derived for the 13 sub-scenarios described in [15]. Following the objective function and the path disjoint constraints for the 13 sub-scenarios, summarized in Table I, we present an integer linear programming (ILP) formulation

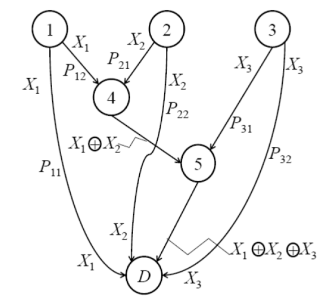


Fig. 1. A possible $3SD$ scenario (scenario 2 sub-scenario 1).

for the scenario presented in Fig. 1, reproduced from [15], in the following.

$$\textbf{min} \sum_{(i,j)\in E} \sum_{m=1}^{2} c_{ij} \times (x_{ij}^{s_1 d,m} + x_{ij}^{s_2 d,m} + x_{ij}^{s_3 d,m}) - \sum_{(i,j)\in E} c_{ij} \times (z_{ij}^1 + z_{ij}^2) \quad (2a)$$

subject to:

$$x_{ij}^{s_a d,b} + x_{ij}^{s_c d,e} \leq 1, \forall (i,j) \in E, \forall (a,b,c,e) \in \mathcal{S} \quad (2b)$$

$$\sum_{j\in V} (x_{ij}^{sd,m} - x_{ji}^{sd,m}) = 1, \forall i, s \in V, \quad i = s,\ s = s_1, s_2, s_3, m = 1, 2 \quad (2c)$$

$$\sum_{j\in V} (x_{ij}^{sd,m} - x_{ji}^{sd,m}) = 0, \quad \forall i, s \in V, \quad i \neq s, d,\ s = s_1, s_2, s_3, m = 1, 2 \quad (2d)$$

$$x_{ij}^{sd,m} = \{0, 1\}, \forall (i,j) \in E, \quad s = s_1, s_2, s_3, m = 1, 2 \quad (2e)$$

$$z_{ij}^1 = \{0, 1\}, \forall (i,j) \in E \quad (2f)$$

$$z_{ij}^1 \leq x_{ij}^{s_1 d,2}, \forall (i,j) \in E \quad (2g)$$

$$z_{ij}^1 \leq x_{ij}^{s_2 d,1}, \forall (i,j) \in E \quad (2h)$$

$$z_{ij}^1 \geq x_{ij}^{s_1 d,2} + x_{ij}^{s_2 d,1} - 1, \forall (i,j) \in E \quad (2i)$$

$$z_{ij}^2 = \{0, 1\}, \forall (i,j) \in E \quad (2j)$$

$$z_{ij}^2 \leq x_{ij}^{s_1 d,2}, \forall (i,j) \in E \quad (2k)$$

$$z_{ij}^2 \leq x_{ij}^{s_3 d,1}, \forall (i,j) \in E \quad (2l)$$

$$z_{ij}^2 \geq x_{ij}^{s_1 d,2} + x_{ij}^{s_3 d,1} - 1, \forall (i,j) \in E. \quad (2m)$$

Eq. (2a) is the objective function which provides optimum minimum cost of employing NC aware disjoint path pairs among three sources and a common destination. Eq. (2b) specifies the path disjoint constraints while flow conservation constraints are specified by the Eqs. (2c)-(2d). Eq. (2c) is the flow conservation constraint at the source nodes. Eq. (2d) describes the flow conservation constraints at the intermediate nodes. The binary routing variable is described by Eq. (2e). Eqs. (2f)-(2m) describe the binary variables, z_{ij}^1 and z_{ij}^2, that determine the proper links to employ NC. These two binary variables can be defined for each scenario following the objective function in the Table I.

The set of vectors $\mathcal{S}$ for the presented ILP model is given as: $\mathcal{S} = \{(1,1,1,2), (2,1,2,2), (1,1,2,1), (1,1,2,2), (1,2,2,2), (1,1,3,1), (1,1,3,2), (2,2,3,2), (2,2,3,1), (1,2,3,2), (2,1,3,2), (3,1,3,2)\}$.

For each of the 13 sub-scenarios (which includes non-NC case) in the $3SD$ case, an optimal mathematical formulation can be derived with the help of Table I. All the sub-scenarios are evaluated using the specific model, and the solution with the minimum total cost is selected as the desired solution.

V. Proposed heuristic routing algorithm

We present our proposed heuristic routing algorithm in order to implement NC aware 1+1 protection routes for all possible source destination pair in a network. According to the given traffic demand matrix, the network with N nodes is divided into N scenarios, where each nodes is chosen as a common destination and k nodes from the remaining ones are the sources, where $2 \leq k \leq N-1$. Assigning NC based 1+1 routes for each of the kSD scenario implies that routing for all possible source destination pairs are considered. We assume that the network has sufficient capacity to serve all the traffic demands. Thus link capacity constraints are not considered.

In the proposed algorithm the ILP models for $2SD$ and $3SD$ scenarios are used. This is because for $kSD, k \geq 4$ scenarios the number of ILPs to be considered increases exponentially, which are not tractable in a practical time [15].

A. Algorithm description

The algorithm is as follows,

- Step 1: Divide the given traffic matrix into N scenarios with k sources and a common destination, where $2 \leq k \leq N-1$. For each of the kSD scenario, repeat step 2.
- Step 2: Select x sources, where x = 2 or 3, out of k sources at a time according to the largest effective gain first policy (to be explained later).
  - Step 2.1: Assign routing to the selected xSD scenario by using the corresponding ILP models presented in [15] (for $x=2$) or in this paper (for $x=3$). Update k as $k = k - x$, which is the remaining number of source nodes.
  - Step 2.2: Select the next x sources from the remaining sources, and assign routing according to step 2.1.
  - Step 2.3: Repeat Step 2.2 until $k-x$ equals 2, 1, or 0. When only 1 source is left, 1+1 protection without NC is applied for that pair. If two sources are left (in case $x=3$), NC based 1+1 protection is applied for this remaining $2SD$ scenario.
- Step 3: The algorithm stops.

B. Policy to select x sources

We describe here the largest effective gain first policy to select x = 2 or 3 sources out of k sources. Prior to describe the policy, we describe the related definitions and notations in the following.

Let φ be the set of source nodes having a common destination, where $s_i \in \varphi, i = 1, 2, \cdots, k$, where $k \geq 2$. A combination of two sources, s_i and s_j, out of k sources, is expressed by $(s_i, s_j) \in \Theta_2, (i<j)$, where Θ_2 is a set of (s_i, s_j). Θ_2 includes $\binom{k}{2}$ combinations of two sources, i.e., pairs. Again, a combination of three sources, s_i, s_j, and s_l, out of k sources, is expressed by $(s_i, s_j, s_l) \in \Theta_3, (i<j<l)$, where Θ_3 is a set of (s_i, s_j, s_l). Θ_3 includes $\binom{k}{3}$ combinations of three sources, i.e., triplets.

Let $\rho_2(s_i, s_j)$ be the product of the NC gain and bandwidth demand for (s_i, s_j). We call $\rho_2(s_i, s_j)$ an effective gain for a $2SD$ scenario, which is expressed by,

$$\rho_2(s_i, s_j) = G_{NC}(s_i, s_j) \times \min(\omega_{s_i d}, \omega_{s_j d}), \tag{3}$$

where $\omega_{s_i d}$ and $\omega_{s_j d}$ are the traffic demands of source nodes s_i and s_j to common destination node d, respectively. $\omega_{s_i d}$ and $\omega_{s_j d}$ are either equal or unequal. In case that two traffic demands are not equal, the effective gain depends on the minimum traffic demand between the two.

Let $\rho_3(s_i, s_j, s_l)$ be the product of the NC gain and bandwidth demand for (s_i, s_j, s_l). We call $\rho_3(s_i, s_j, s_l)$ an effective gain for a $3SD$ scenario, which is expressed by,

$$\rho_3(s_i, s_j, s_l) = G_{NC}(s_i, s_j, s_l) \times \min(\omega_{s_i d}, \omega_{s_j d}, \omega_{s_l d}), \tag{4}$$

where $\omega_{s_i d}$, $\omega_{s_j d}$, and $\omega_{s_j d}$ are the traffic demands of source nodes s_i, s_j, and s_l to common destination node d, respectively. $\omega_{s_i d}$, $\omega_{s_j d}$, and $\omega_{s_l d}$ are either equal or unequal.[1] In case that three traffic demands are not equal, the effective gain depends on the minimum traffic demand between the three.

1) Largest effective gain first policy:

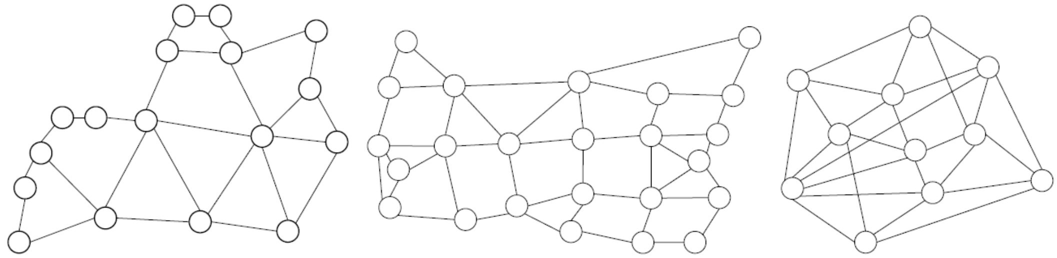
- Step 1: For all $(s_i, s_j) \in \Theta_2$, compute $\rho_2(s_i, s_j)$, or for all $(s_i, s_j, s_l) \in \Theta_3$ compute $\rho_3(s_i, s_j, s_l)$. If we compute $\rho_2(s_i, s_j)$, then go to Step 2. If $\rho_3(s_i, s_j, s_l)$ is computed, go to Step 3.
- Step 2: REPEAT
  - Step 2.1: Select a pair $(s_i, s_j) \in \Theta_2$ with the highest $\rho_2(s_i, s_j)$.
  - Step 2.2: Remove (s_i, s_j) and all other pairs that include either s_i or s_j from Θ_2.

  UNTIL Θ_2 is empty.
- Step 3: REPEAT
  - Step 3.1: Select a pair $(s_i, s_j, s_l) \in \Theta_3$ with the highest $\rho_3(s_i, s_j, s_l)$.
  - Step 2.2: Remove (s_i, s_j, s_l) and all other triplets that include either s_i or s_j or s_l from Θ_3.

  UNTIL Θ_3 is empty.

At the end we select a set of $\lfloor \frac{k}{2} \rfloor$ pairs or $\lfloor \frac{k}{3} \rfloor$ triplets with the highest effective gain in each of the case. The number of times we compute the effective gains for pairs $\in \Theta_2$ is expressed as

[1] In this paper we consider equal traffic demands for all the pairs. However, our presented model for $3SD$ scenarios is applicable for unequal traffic demands. We have to add the related constraints.



(a) Germany 17 network (b) US IP Backbone network (c) COST 239 network

Fig. 2. Evaluated networks

$W_2(k)$,[2] which is given by,

$$W_2(k) = \begin{cases} \frac{k(k-1)}{2}, & \text{if } k \text{ is even} \\ \frac{k(k-1)(k-2)}{2}, & \text{if } k \text{ is odd.} \end{cases} \tag{5}$$

The number of times we compute the effective gains for triplets $\in \Theta_3$ is expressed as $W_3(k)$, which is given by,

$$W_3(k) = \begin{cases} \frac{k(k-1)(k-2)}{6}, & \text{if } k = 3 \times z, z\text{=integer} \\ \frac{k(k-1)(k-2)(k-3)}{6}, & \text{if } k = 3 \times z + 1 \\ \frac{k(k-1)(k-2)(k-3)(k-4)}{12}, & \text{if } k = 3 \times z + 2. \end{cases} \tag{6}$$

When we select a pair at a time, this policy has the computational complexity of the order of $O(k^2)$ when k is even, and $O(k^3)$ when Y is odd. When we select a triplet at a time, this policy has the computational complexity of the order of $O(k^3)$ when $k = 3 \times z$, $O(k^4)$ when $k = 3 \times z + 1$, and $O(k^5)$ when $k = 3 \times z + 2$.

VI. Results and discussion

We compute twice the total path cost of implementing NC based 1+1 protection for all possible source destination pairs by using our proposed heuristic algorithm. In the first computation the $2SD$ scenarios, and in the second computation the $3SD$ scenarios are selected one by one according to the largest effective gain first policy. We compared the two total path costs, achieved by our proposed algorithm, to the same total cost when conventional 1+1 protection technique without NC is employed. We evaluated the above mentioned total costs in the three networks as shown in Fig. 2. The mathematical models are solved by using CPLEX®[16] as the LP solver. In the evaluation all nodes in the network are assumed to have NC capability. In the network topologies, link utilization cost is unity, and the traffic demands are the same for all source-destination pairs.

From the results of Fig. 3 it is observed that, the proposed algorithm with the $3SD$ scenario achieves the highest resource saving effect in our examined networks. Figure 3 illustrates that almost 15% resource saving is achieved in the COST 239 network by selecting three sources at a time with the largest effective gain first policy. In the COST 239 network

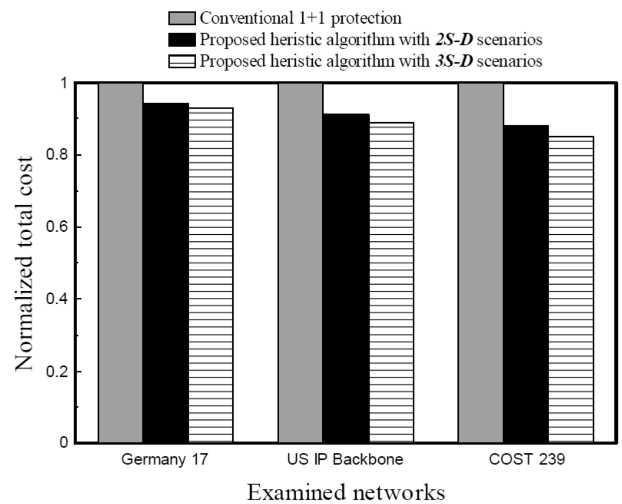


Fig. 3. Comparison of total costs. The costs are normalized w.r.to the total cost of 1+1 protection.

the smallest node degree is four, and all the source destination pairs in this network can avail the resource saving advantage of the NC based 1+1 protection technique. In the US IP Backbone and Germany 17 networks, there are a number of pairs to which NC based 1+1 protection technique is not applicable, because for those pairs the destination node degree is only two. Thus some of the $2SD$ and $3SD$ scenarios cannot obtain the benefit of resource saving due to NC in these two networks. This is the reason of lower resource saving in the US IP backbone and Germany 17 networks.

VII. Conclusions

This paper has proposed a heuristic routing algorithm to design instantaneous recovery protection routes for all possible source destination pairs by provisioning network coding (NC) based 1+1 protection technique. In the proposed algorithm, a network with N nodes is divided into N scenarios, where each node is chosen as a common destination, and k nodes from the remaining ones are selected as sources, where $2 \leq k \leq N-1$. A mathematical programming solution for NC based 1+1 routing in scenarios with k sources and a common destination (kSD) is an intractable problem for large k values. In the proposed algorithm, for each of the divided kSD scenario, out of k sources we choose either two or three sources at a

[2]When k is even, $W_2(k)$ is derived from, $W_2(k) = \binom{k}{2}$. When k is odd, $W_2(k)$ is derived from $W_2(k) = k \times W_2(k-1) = k \times \binom{k-1}{2}$. A similar computation is applicable to $W_3(k)$.

TABLE I

OBJECTIVE FUNCTIONS AND CONSTRAINTS

Scenario (SC) No.	Objective Function	Disjoint Path Pairs
SC 1	$\sum_{(i,j)\in E}\sum_{m=1}^{2} C_{ij}\times(x_{ij}^{s_1d,m}+x_{ij}^{s_2d,m}+x_{ij}^{s_3d,m})$	$P_{11}\&P_{12}$, $P_{21}\&P_{22}$, $P_{31}\&P_{32}$
SC 2 Sub-SC 1	$\sum_{(i,j)\in E}\sum_{m=1}^{2} C_{ij}\times(x_{ij}^{s_1d,m}+x_{ij}^{s_2d,m}+x_{ij}^{s_3d,m})-\sum_{(i,j)\in E}(C_{ij}\times x_{ij}^{s_1d,2}\times x_{ij}^{s_2d,1}+C_{ij}\times x_{ij}^{s_1d,2}\times x_{ij}^{s_3d,1})$	$P_{11}\&P_{22}$, $P_{11}\&P_{32}$, $P_{11}\&P_{12}$, $P_{11}\&P_{21}$, $P_{11}\&P_{31}$, $P_{22}\&P_{32}$, $P_{22}\&P_{12}$, $P_{22}\&P_{21}$, $P_{22}\&P_{31}$, $P_{32}\&P_{12}$, $P_{32}\&P_{21}$, $P_{32}\&P_{31}$
SC 2 Sub-SC 2	$\sum_{(i,j)\in E}\sum_{m=1}^{2} C_{ij}\times(x_{ij}^{s_1d,m}+x_{ij}^{s_2d,m}+x_{ij}^{s_3d,m})-\sum_{(i,j)\in E}(C_{ij}\times x_{ij}^{s_1d,2}\times x_{ij}^{s_3d,1}+C_{ij}\times x_{ij}^{s_2d,1}\times x_{ij}^{s_3d,1})$	$P_{11}\&P_{22}$, $P_{11}\&P_{32}$, $P_{11}\&P_{12}$, $P_{11}\&P_{21}$, $P_{11}\&P_{31}$, $P_{22}\&P_{32}$, $P_{22}\&P_{12}$, $P_{22}\&P_{21}$, $P_{22}\&P_{31}$, $P_{32}\&P_{12}$, $P_{32}\&P_{21}$, $P_{32}\&P_{31}$
SC 2 Sub-SC 3	$\sum_{(i,j)\in E}\sum_{m=1}^{2} C_{ij}\times(x_{ij}^{s_1d,m}+x_{ij}^{s_2d,m}+x_{ij}^{s_3d,m})-\sum_{(i,j)\in E}(C_{ij}\times x_{ij}^{s_2d,1}\times x_{ij}^{s_3d,1}+C_{ij}\times x_{ij}^{s_1d,2}\times x_{ij}^{s_2d,1})$	$P_{11}\&P_{22}$, $P_{11}\&P_{32}$, $P_{11}\&P_{12}$, $P_{11}\&P_{21}$, $P_{11}\&P_{31}$, $P_{22}\&P_{32}$, $P_{22}\&P_{12}$, $P_{22}\&P_{21}$, $P_{22}\&P_{31}$, $P_{32}\&P_{12}$, $P_{32}\&P_{21}$, $P_{32}\&P_{31}$
SC 3 Sub-SC 1	$\sum_{(i,j)\in E}\sum_{m=1}^{2} C_{ij}\times(x_{ij}^{s_1d,m}+x_{ij}^{s_2d,m}+x_{ij}^{s_3d,m})-\sum_{(i,j)\in E}(C_{ij}\times x_{ij}^{s_1d,2}\times x_{ij}^{s_2d,1}+C_{ij}\times x_{ij}^{s_2d,2}\times x_{ij}^{s_3d,1})$	$P_{11}\&P_{32}$, $P_{11}\&P_{12}$, $P_{11}\&P_{21}$, $P_{11}\&P_{22}$, $P_{11}\&P_{31}$, $P_{32}\&P_{12}$, $P_{32}\&P_{21}$, $P_{32}\&P_{22}$, $P_{32}\&P_{31}$, $P_{12}\&P_{22}$, $P_{12}\&P_{31}$, $P_{21}\&P_{22}$, $P_{21}\&P_{31}$
SC 3 Sub-SC 2	$\sum_{(i,j)\in E}\sum_{m=1}^{2} C_{ij}\times(x_{ij}^{s_1d,m}+x_{ij}^{s_2d,m}+x_{ij}^{s_3d,m})-\sum_{(i,j)\in E}(C_{ij}\times x_{ij}^{s_1d,1}\times x_{ij}^{s_2d,1}+C_{ij}\times x_{ij}^{s_1d,2}\times x_{ij}^{s_3d,1})$	$P_{22}\&P_{32}$, $P_{22}\&P_{11}$, $P_{22}\&P_{21}$, $P_{22}\&P_{12}$, $P_{22}\&P_{31}$, $P_{32}\&P_{11}$, $P_{32}\&P_{21}$, $P_{32}\&P_{12}$, $P_{32}\&P_{31}$, $P_{11}\&P_{12}$, $P_{11}\&P_{31}$, $P_{21}\&P_{12}$, $P_{21}\&P_{31}$
SC 3 Sub-SC 3	$\sum_{(i,j)\in E}\sum_{m=1}^{2} C_{ij}\times(x_{ij}^{s_1d,m}+x_{ij}^{s_2d,m}+x_{ij}^{s_3d,m})-\sum_{(i,j)\in E}(C_{ij}\times x_{ij}^{s_1d,2}\times x_{ij}^{s_3d,1}+C_{ij}\times x_{ij}^{s_2d,2}\times x_{ij}^{s_3d,2})$	$P_{11}\&P_{21}$, $P_{11}\&P_{12}$, $P_{11}\&P_{31}$, $P_{11}\&P_{22}$, $P_{11}\&P_{32}$, $P_{21}\&P_{12}$, $P_{21}\&P_{31}$, $P_{21}\&P_{22}$, $P_{21}\&P_{32}$, $P_{12}\&P_{22}$, $P_{12}\&P_{32}$, $P_{31}\&P_{22}$, $P_{31}\&P_{32}$
SC 4 Sub-SC 1	$\sum_{(i,j)\in E}\sum_{m=1}^{2} C_{ij}\times(x_{ij}^{s_1d,m}+x_{ij}^{s_2d,m}+x_{ij}^{s_3d,m})-\sum_{(i,j)\in E}(C_{ij}\times x_{ij}^{s_1d,2}\times x_{ij}^{s_2d,1})$	$P_{11}\&P_{12}$, $P_{11}\&P_{21}$, $P_{11}\&P_{22}$, $P_{11}\&P_{32}$, $P_{31}\&P_{12}$, $P_{31}\&P_{21}$, $P_{31}\&P_{22}$, $P_{31}\&P_{32}$, $P_{12}\&P_{32}$,, $P_{12}\&P_{22}$, $P_{21}\&P_{22}$, $P_{21}\&P_{32}$
SC 4 Sub-SC 2	$\sum_{(i,j)\in E}\sum_{m=1}^{2} C_{ij}\times(x_{ij}^{s_1d,m}+x_{ij}^{s_2d,m}+x_{ij}^{s_3d,m})-\sum_{(i,j)\in E}(C_{ij}\times x_{ij}^{s_1d,1}\times x_{ij}^{s_3d,1})$	$P_{11}\&P_{12}$, $P_{11}\&P_{21}$, $P_{11}\&P_{22}$, $P_{11}\&P_{32}$, $P_{31}\&P_{12}$, $P_{31}\&P_{21}$, $P_{31}\&P_{22}$, $P_{31}\&P_{32}$, $P_{12}\&P_{32}$, $P_{12}\&P_{22}$, $P_{21}\&P_{22}$, $P_{21}\&P_{32}$
SC 4 Sub-SC 3	$\sum_{(i,j)\in E}\sum_{m=1}^{2} C_{ij}\times(x_{ij}^{s_1d,m}+x_{ij}^{s_2d,m}+x_{ij}^{s_3d,m})-\sum_{(i,j)\in E}(C_{ij}\times x_{ij}^{s_2d,2}\times x_{ij}^{s_3d,2})$	$P_{11}\&P_{12}$, $P_{11}\&P_{21}$, $P_{11}\&P_{22}$, $P_{11}\&P_{32}$, $P_{31}\&P_{12}$, $P_{31}\&P_{21}$, $P_{31}\&P_{22}$, $P_{31}\&P_{32}$, $P_{12}\&P_{32}$, $P_{12}\&P_{22}$, $P_{21}\&P_{22}$, $P_{21}\&P_{32}$
SC 5 Sub-SC 1	$\sum_{(i,j)\in E}\sum_{m=1}^{2} C_{ij}\times(x_{ij}^{s_1d,m}+x_{ij}^{s_2d,m}+x_{ij}^{s_3d,m})-\sum_{(i,j)\in E}(C_{ij}\times x_{ij}^{s_1d,2}\times x_{ij}^{s_2d,1})$	$P_{11}\&P_{12}$, $P_{11}\&P_{21}$, $P_{11}\&P_{22}$, $P_{11}\&P_{31}$, $P_{11}\&P_{32}$, $P_{12}\&P_{22}$, $P_{12}\&P_{31}$, $P_{12}\&P_{32}$, $P_{21}\&P_{22}$, $P_{21}\&P_{31}$, $P_{21}\&P_{32}$, $P_{22}\&P_{31}$, $P_{22}\&P_{32}$, $P_{31}\&P_{32}$
SC 5 Sub-SC 2	$\sum_{(i,j)\in E}\sum_{m=1}^{2} C_{ij}\times(x_{ij}^{s_1d,m}+x_{ij}^{s_2d,m}+x_{ij}^{s_3d,m})-\sum_{(i,j)\in E}(C_{ij}\times x_{ij}^{s_1d,2}\times x_{ij}^{s_3d,1})$	$P_{11}\&P_{12}$, $P_{11}\&P_{21}$, $P_{11}\&P_{22}$, $P_{11}\&P_{31}$, $P_{11}\&P_{32}$, $P_{12}\&P_{21}$, $P_{12}\&P_{22}$, $P_{12}\&P_{32}$, $P_{21}\&P_{22}$, $P_{21}\&P_{31}$, $P_{21}\&P_{32}$, $P_{22}\&P_{31}$, $P_{22}\&P_{32}$, $P_{31}\&P_{32}$
SC 5 Sub-SC 3	$\sum_{(i,j)\in E}\sum_{m=1}^{2} C_{ij}\times(x_{ij}^{s_1d,m}+x_{ij}^{s_2d,m}+x_{ij}^{s_3d,m})-\sum_{(i,j)\in E}(C_{ij}\times x_{ij}^{s_2d,2}\times x_{ij}^{s_3d,1})$	$P_{11}\&P_{12}$, $P_{11}\&P_{21}$, $P_{11}\&P_{22}$, $P_{11}\&P_{31}$, $P_{11}\&P_{32}$, $P_{12}\&P_{21}$, $P_{12}\&P_{22}$, $P_{12}\&P_{31}$, $P_{12}\&P_{32}$, $P_{21}\&P_{22}$, $P_{21}\&P_{31}$, $P_{21}\&P_{32}$, $P_{22}\&P_{32}$, $P_{31}\&P_{32}$

time according to the largest effective gain first policy, and then routing is assigned to the selected $2SD$ or $3SD$ scenario using the respective mathematical formulations. We compared the total path costs of NC based 1+1 protection for all possible source destination pairs, obtained by our proposed heuristic algorithm, with that of the conventional 1+1 protection technique (without NC). Almost 15% resource saving w.r.to 1+1 protection is achieved in our examined networks. The proposed heuristic algorithm with three sources selected at a time achieves a higher resource saving as compared to that with two sources selected at a time.

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APPENDIX

The objective functions and path disjoint constraints for the 13 possible $3SD$ scenarios are summarized in Table I.